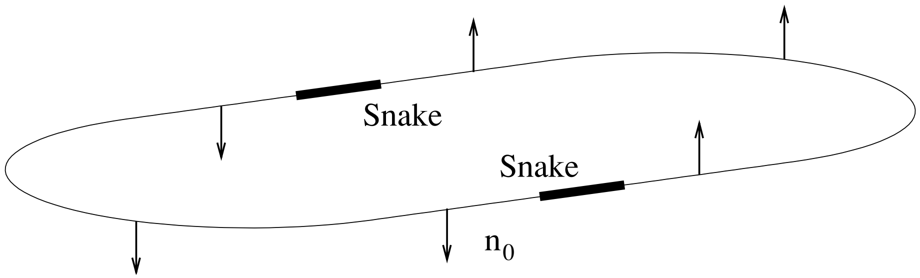
Two Siberian Snakes Configuration Properties

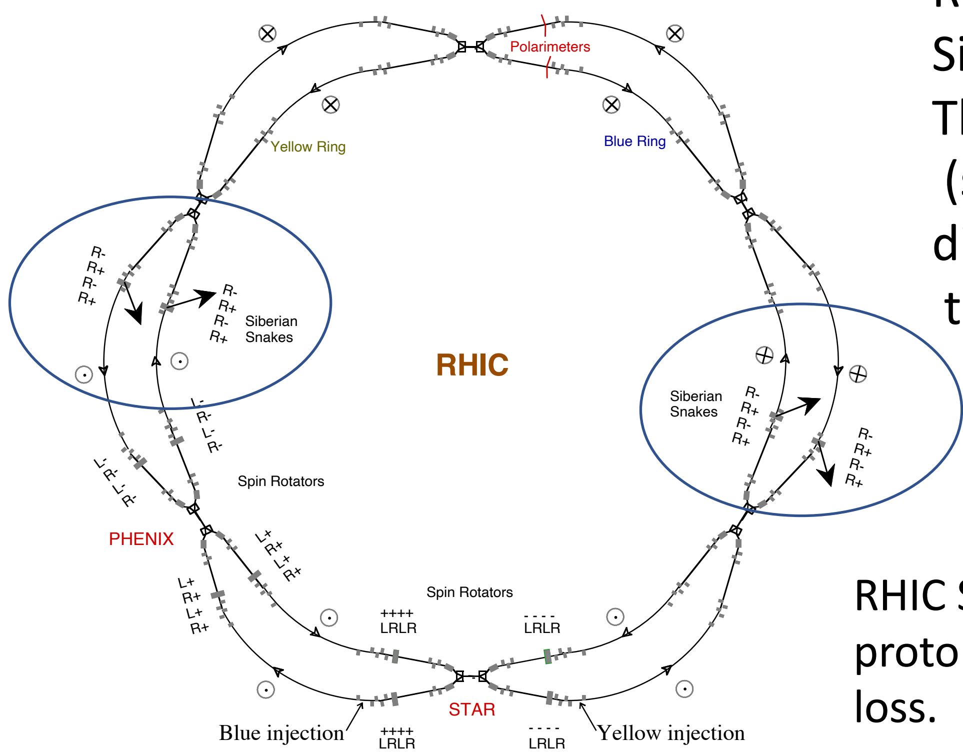


a) Stable spin direction $\mathbf{n_0}$ is vertical in the ring arcs, pointing up in one half, and down on another.
b) Spin tune is independent on energy and defined by the Snake axis orientations:

$$\nu_{sp} = \frac{2(\alpha_{s1} - \alpha_{s2})}{2\pi} = \frac{(\alpha_{s1} - \alpha_{s2})}{\pi}$$

Thus, for instance, to get the spin tune equal to 0.5, the Snake axes should be at 90 degree angle to each other.

Siberian Snakes in RHIC



RHIC employs the configuration with two Siberian Snakes in each ring.
The Snake axes are at 45 and 135 degrees (symmetrical around longitudinal direction),
thus giving the spin tune 0.5

RHIC Snakes allowed for acceleration of polarized protons up to 255 GeV with minimal polarization loss.

High order resonances (Snake resonances) are pronounced at higher energies and has to be avoided:

$$\nu_{sp} = N + m \cdot Q_y$$

Even Number of Snakes

As the resonance strength increases with the beam energy, the inceased number of Snake may need to be employed in future accelerators. Now let's consider 2N Snakes distributed around the ring at azimuth $\theta_1, \theta_2 ... \theta_{2N}=2\pi$.
Each Snake is characterized by its own Snake axis angle α_{si}.

<u>Calculation of one turn matrix is a nice exercise for the Homework.</u> Here matrices V present spin rotation around vertical axis in the ring arcs between the Snakes.

$$
\begin{aligned}
M_{turn} &= M_{sn,2N} V_{\theta_{2N},\theta_{2N-1}} M_{sn,2N-1} V_{\theta_{2N-1},\theta_{2N-2}} \ldots M_{sn,1} V_{\theta_1,\theta_0} \\
&= \ldots \\
&= V(\phi) = \exp(-i\sigma_3 \frac{\phi}{2})
\end{aligned}
$$

Thus, in one turn the spin rotates around the vertical axis by an angle which depends on Snake locations and Snake axis orientations:

$$
\phi = G\gamma \sum_{i=1}^{2N} (-1)^{i-1} (\theta_i - \theta_{i-1}) + 2 \sum_{i=1}^{N} (\alpha_{s,2i} - \alpha_{s,2i-1})
$$

Properties of Configuration with Even Number of Snakes

Spin tune then is:

$$\nu_{sp} = \underbrace{\frac{G\gamma}{2\pi}\sum_{i=1}^{2N}(-1)^{i-1}(\theta_i - \theta_{i-1})}_{} + \frac{1}{\pi}\sum_{i=1}^{N}(\alpha_{s,2i} - \alpha_{s,2i-1})$$

Main rule: place the Snakes in such locations that this term becomes 0.
The spin tune is independent on energy.

- If spin tune is independent on energy the spin resonance conditions are avoided during the acceleration process.
- By selecting proper orientations of Snake axes one can choose the value of the spin tune. Common approach to have it at 0.5.
- Stable spin direction $\mathbf{n_0}$ is vertical in arcs. Each Snake switches $\mathbf{n_0}$ from up to down, and vice verse.

Spin Rotators in Accelerators

- Natural orientation of stable spin direction in accelerators is vertical. Particle physics experiments often require a specific polarization orientation (often, longitudinal) in experimental detectors.
- In a circular accelerator usually a pair of spin rotators is installed, where the second rotator restored the polarization orientation to vertical.
- Spin rotators also often used at low energies to convert the beam polarization produced by the particle source to a wanted orientation.

Realization of Snakes and Spin Rotators

Solenoidal Snake

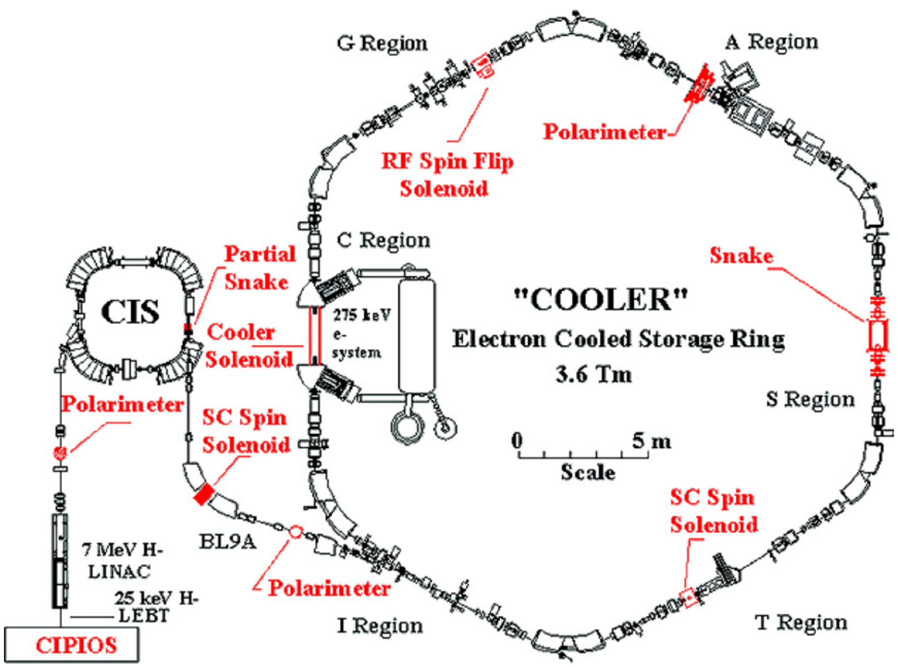
- The required field for 180 degree spin rotation in solenoidal magnet:

For electrons: $B_{sol}L = 10.47 \cdot p(GeV/c)$

For protons: $B_{sol}L = 3.75 \cdot p(GeV/c)$

- Beam closed orbit is no affected!
- But the betatron coupling introduced by the solenoid require compensation
- Good for full snake applications below 10-20 GeV beam energy

Solenoidal Siberian snake at IUCF



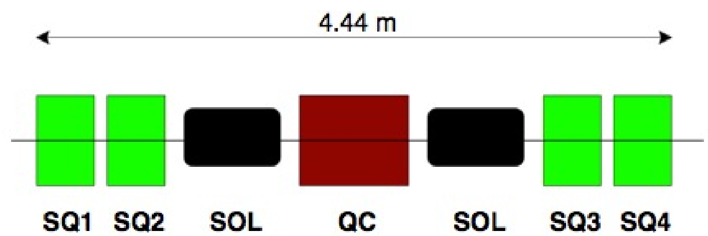
- Solenoidal Snake was used in IUCF 500 MeV Cooler Ring for proof-of-principle experiments (1989-1997)
- The Snake: 2 T*m solenoid
- First observation that depolarizing resonances were overcome by the Snake.
- First observation of Snake resonances.

Solenoidal Siberian Snake Compensation Scheme

Applications of solenoidal Snakes to create longitudinal polarization on internal targets in AmPS and MIT-Bates SHR were shown on slide 27. Beam energy ~0.7-1 GeV.

Superconducting Solenoidal Snake in MIT-Bates

Betatron coupling is compensated using the system of normal and skew-quadrupoles. .



Strong longitudinal field magnets: total integrated field up to 9.5 Tm

Dividing the solenoid in two halves with Interspersed quadrupoles presents standard approach for solenoidal compensation

Simple Solenoidal Spin Rotator

- Combination of a solenoidal magnet and horizontally bending dipole
- Transformation matrix for converting the vertical spin to longitudinal one is very simple:

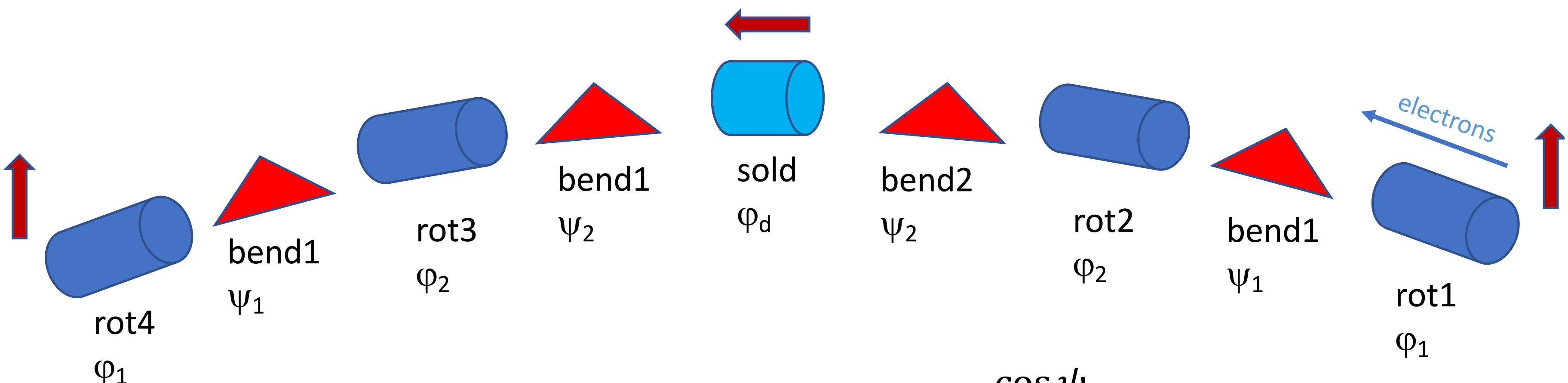
$$M_{rot} = \exp(-i\pi\sigma_3/4)\exp(-i\pi\sigma_2/4)$$

It requires 90 degree spin rotation by solenoid accompanied by 90 degree spin rotation by the horizontal bend

- Can be used also in transfer lines to convert the longitudinal polarization to the vertical one
  SLC spin rotator
- Deficiency: works at one particular energy
- One of Homeworks ask you to evaluate this type of the rotator.

Solenoidal Rotator for EIC

- To operate in wide energy range the rotator scheme must use at least two solenoidal insertions
- General rotator scheme for EIC electrons (5-18 GeV):



Relations for longitudinal polarization:

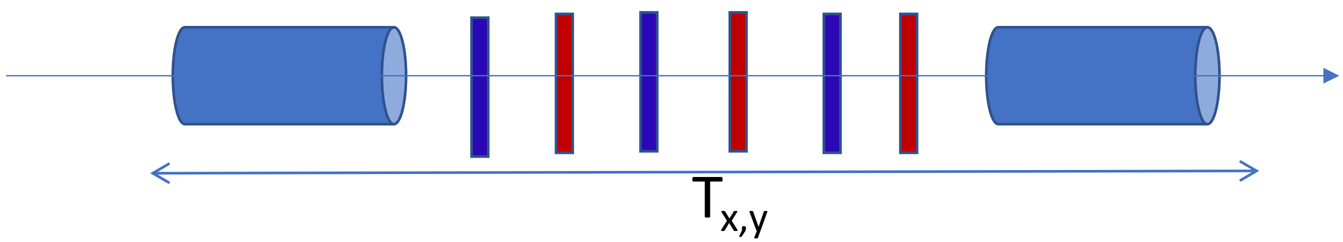
$$\tan \varphi_1 = \pm \frac{\cos \psi_2}{\sqrt{-\cos(\psi_1 + \psi_2)\cos(\psi_1 - \psi_2)}}$$

$$\cos \varphi_2 = \cot \psi_1 \ \cot \psi_2$$

Example of Solenoid insertion for spin rotator

Optics of the solenoid insertion in electron ring must realize two independent conditions:

- Betatron coupling has to be compensated by the use of normal and skew quadrupoles
- Specific spin matching conditions has to satisfied to minimize depolarization (Next week lecture!)



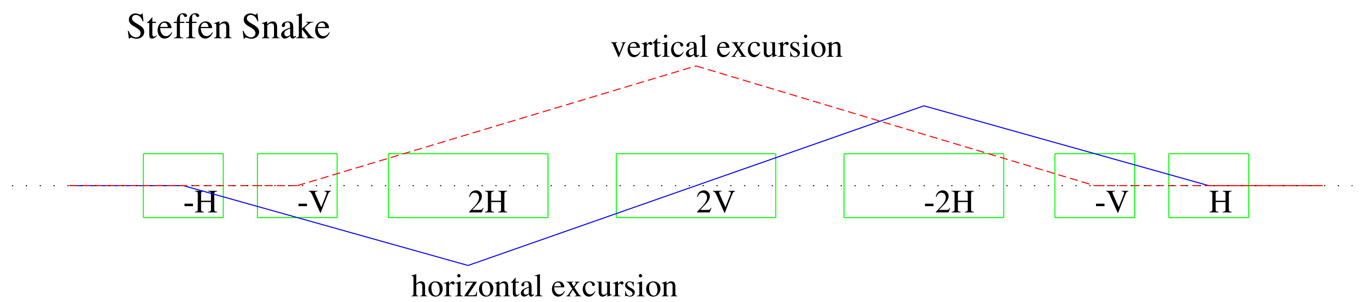
For a betatron spin-matched and fully decoupled solenoidal insertion the horizontal and vertical transport matrices must have following forms:

$$T_X = \begin{pmatrix} -\cos(\varphi) & -\frac{2}{K_s}\sin(\varphi) \\ \frac{K_s}{2}\sin(\varphi) & -\cos(\varphi) \end{pmatrix}; \quad T_Y = -T_X = \begin{pmatrix} \cos(\varphi) & \frac{2}{K_s}\sin(\varphi) \\ -\frac{K_s}{2}\sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

$$K_s = \frac{B_s}{B\rho} \qquad \varphi = (1+a)K_s L$$

Siberian Snake Based on Dipole Magnets

- Alternating vertical and horizontal bends. Proposed by K. Steffen.



- Special field symmetry with respect to the Snake center:
  - vertical field (H-bend) – anti-symmetric
  - horizontal field (V-bend) – symmetric

automatically gives the Snake axis in horizontal plane and the beam orbit restored.

Steffen's Snake Parameter Choice

Spin transformation matrix of Steffen's Snake:

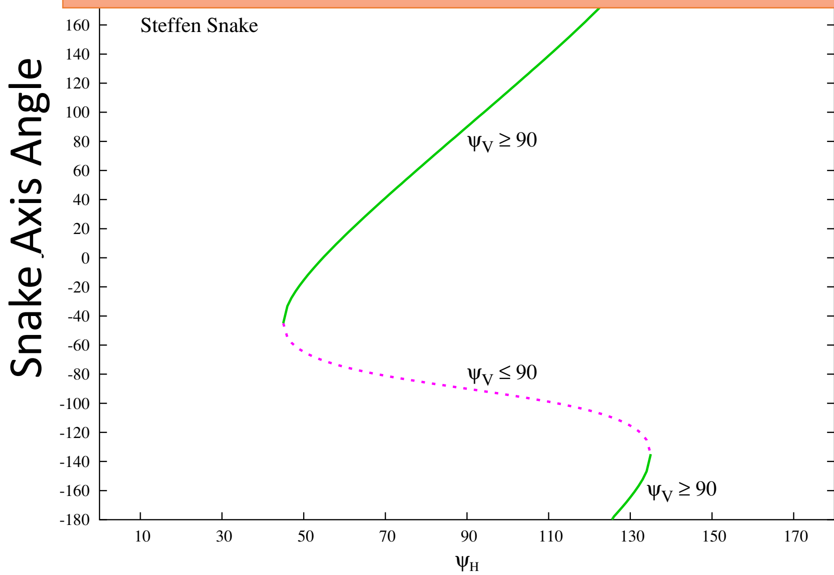
$$M_{\text{Steffen}} = \mathrm{e}^{-\mathrm{i}\psi_H\sigma_3/2}\,\mathrm{e}^{\mathrm{i}\psi_V\sigma_1/2}\,\mathrm{e}^{\mathrm{i}\psi_H\sigma_3}\,\mathrm{e}^{-\mathrm{i}\psi_V\sigma_1}\,\mathrm{e}^{-\mathrm{i}\psi_H\sigma_3}\,\mathrm{e}^{\mathrm{i}\psi_V\sigma_1/2}\,\mathrm{e}^{\mathrm{i}\psi_H\sigma_3/2}.$$

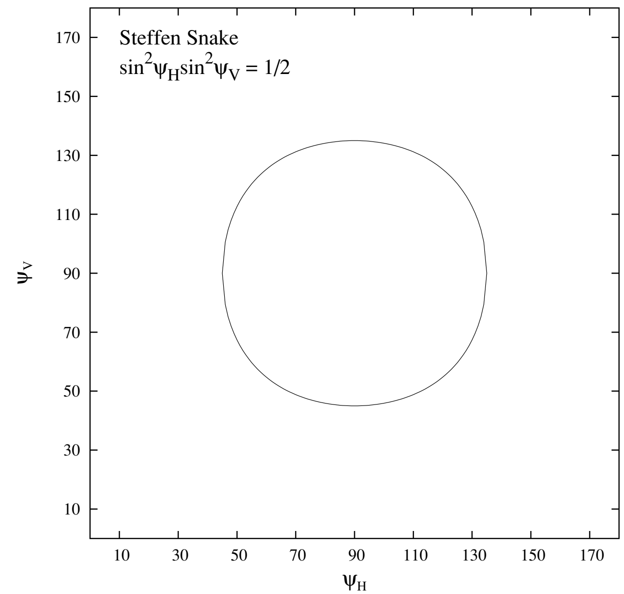
From here for Full Snake one gets:

$$\sin^2\psi_H \sin^2\psi_V = \frac{1}{2}.$$

H-V relation plot

Any Snake axis direction may be selected depending on the choice of fields





Properties of Steffen's Snake

- Required fields scale inversely proportionally to particle velocity
  thus practically do not depend on the beam energy for relativistic beams

- The orbit excursion changes inversely proportionally to the particle energy
  typically exceeds 5cm for the energies below 40 GeV

- Required integrated magnetic field is generally in the range 15-35 Tm, depending on the orientation of the Snake axis.

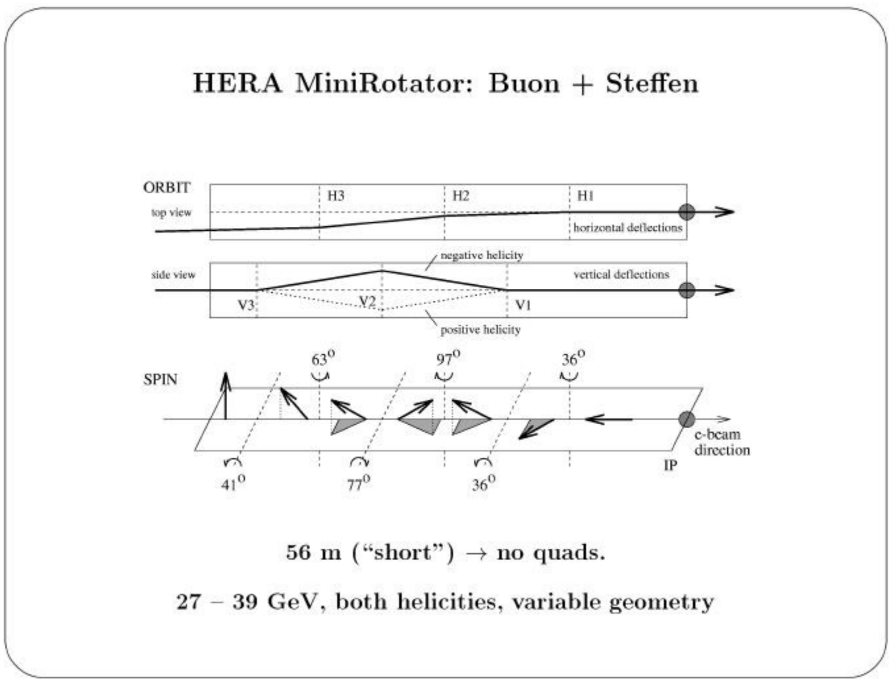
Dipole Magnet Snake vs Solenoidal Snake

- Unpleasant feature of the dipole magnet Snake, as compared with solenoidal snake/rotators, is that the beam orbit significantly distorted inside the Snake.
- Below 20 GeV energy the orbit excursion reaches tens of centimeters.
- But a solenoidal Snake would require very large field integral (hundreds T*m) at the energies above 20 GeV.

- Thus, use:
  - dipole field Snakes/rotators at the energies above 10 GeV
  - solenoidal Snakes/rotators below 20 GeV.

HERA spin rotator

- HERA was the first e-p collider, operated with 27.5 GeV electrons and 820 (920) GeV protons.
- The spin rotators were implemented for electron beam to produce longitudinal polarization at the experimental detectors



Sequence of horizontal and vertical bends:

(V1, H3, V2, H2, V3, H1).

Vertical orbit is restored:
V3 = -(V1+V2)

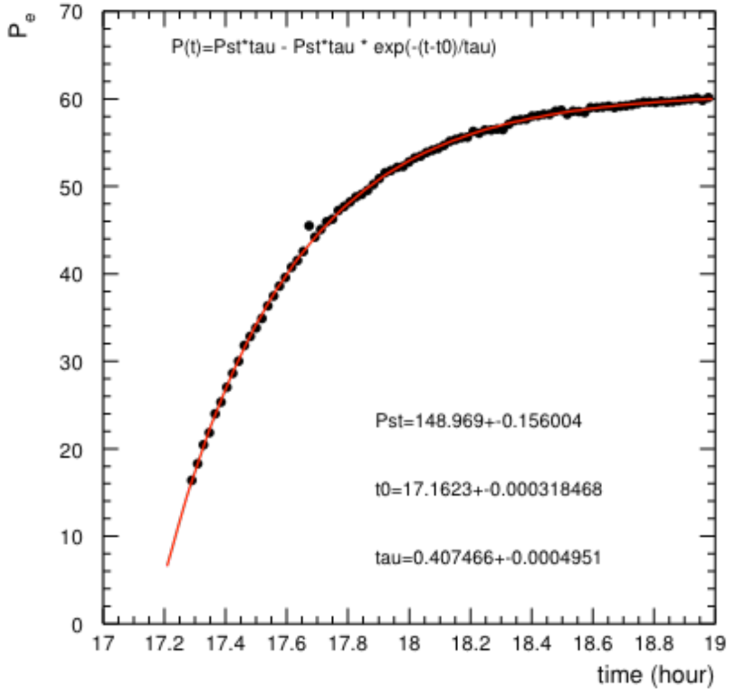
But horizontally there is a small net bending angle coming from the rotator.

Rotator insertion optics was designed to satisfy the spin matching conditions.

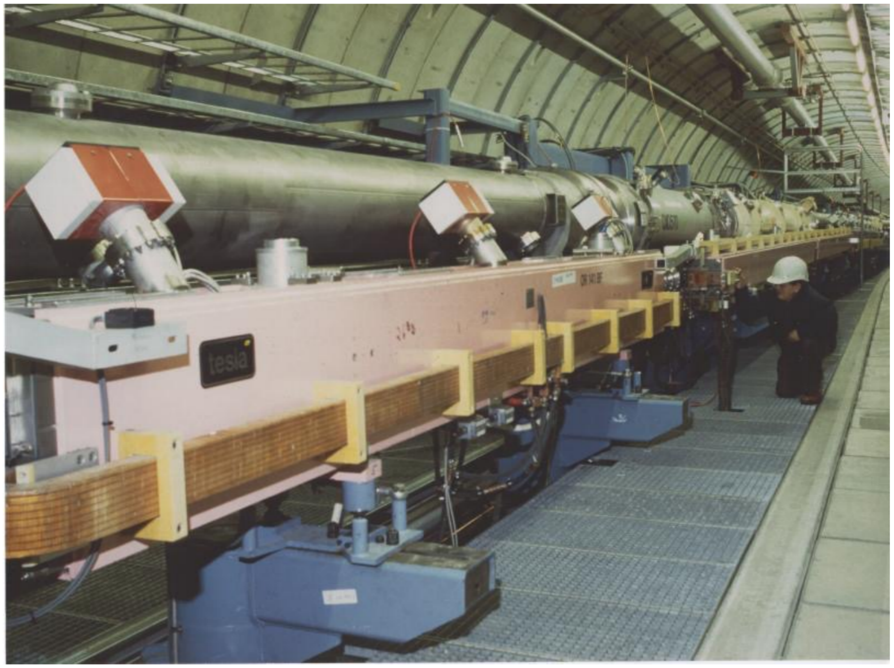
HERA Rotators

Highest energy application of the spin rotators in electron accelerators

Polarization build-up with rotators in HERA



Implemented as normal conducting magnets



Inconvenient feature of this design:
Changing polarization direction at the experiments required vertical movement of the magnets

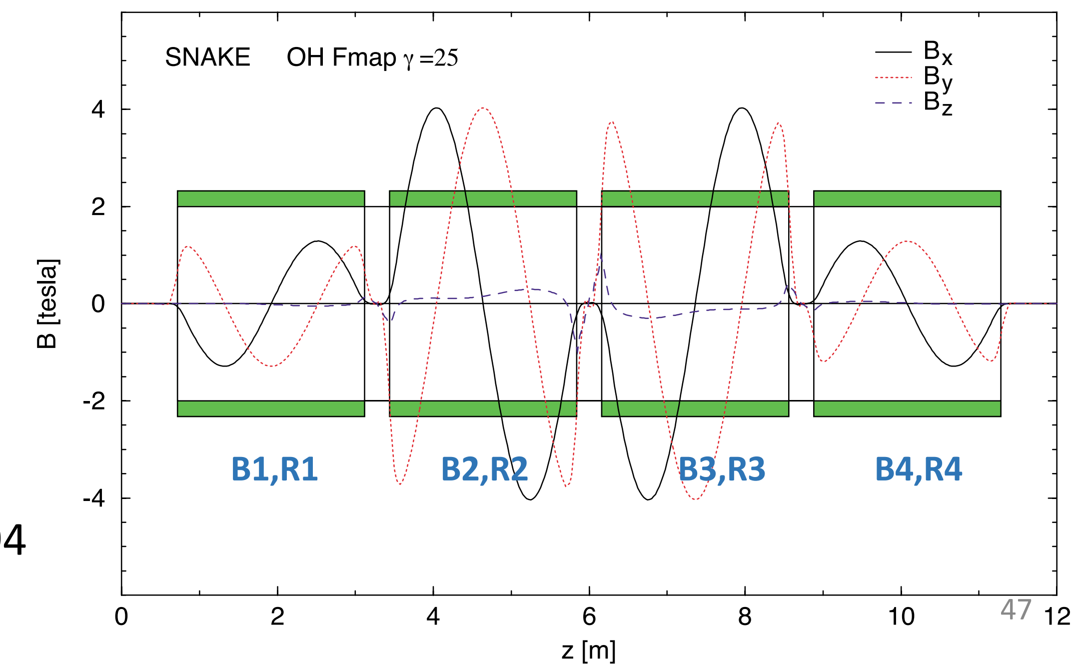
Helical Siberian Snake

- Continuous axis snake can be created with **four full twist helical magnets**, having vertically oriented field at the entrance of each helix.
- Symmetry conditions for fields and helicities:

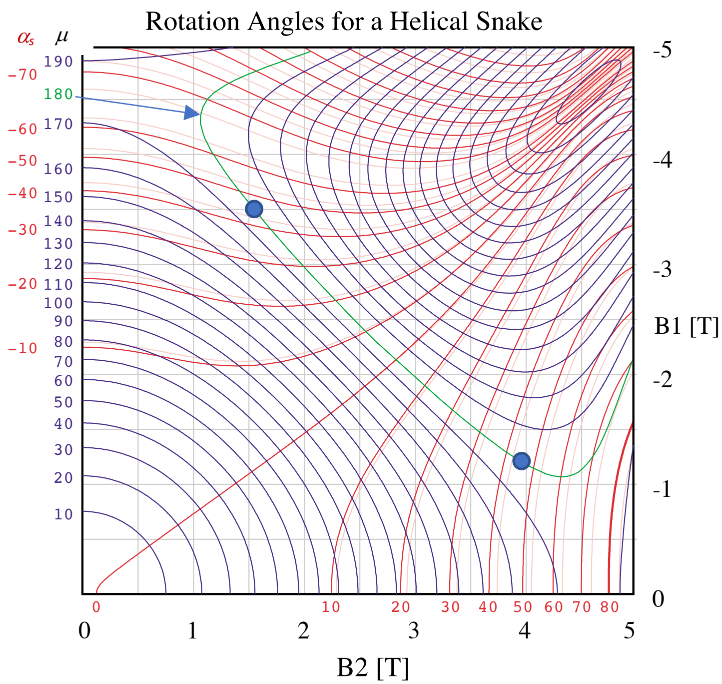
  B1=-B4; B2 = -B3; R1 = R4; R2 =R3

Field configuration in RHIC helical snake

Ptitsyn-Shatunov, 1994



Helical Snake Parameter Choice



Parameter plot for RHIC Snake.

μ is the spin rotation angle
For full Snake $\mu = 180^o$ is needed (green curve).

α_s is the Snake axis angle. (On this plot it is accounted from the longitudinal axis!)

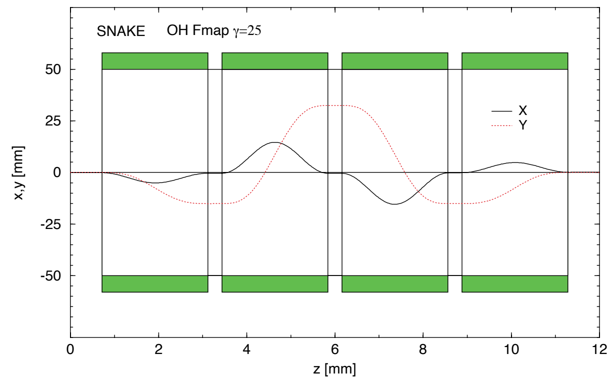
From this plot one can find helical fields (B1, B2) required to achieve given μ and α_s

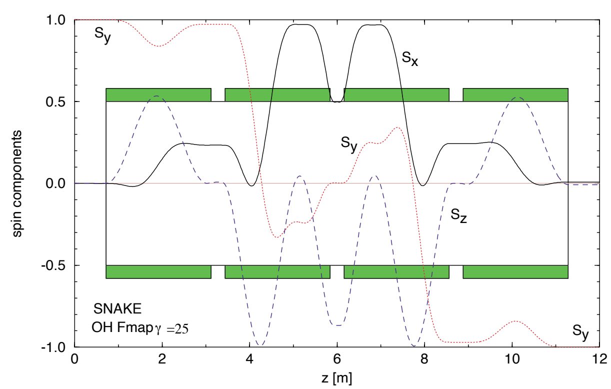
Natural choice, applied in RHIC is to have one Snake axis at +45 degrees with other Snake axis oriented at -45 degrees.
Blue circles show operational parameter points for RHIC Snakes.

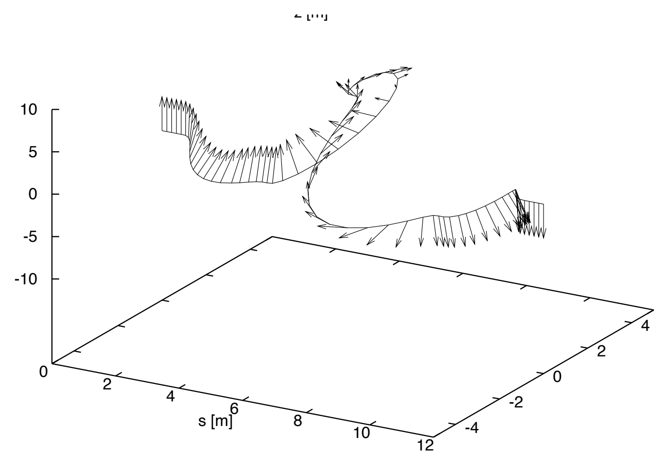
Required total field integral ~25-30 Tm

RHIC Helical Snake

Spin and orbit evolution
through RHIC helical Snake



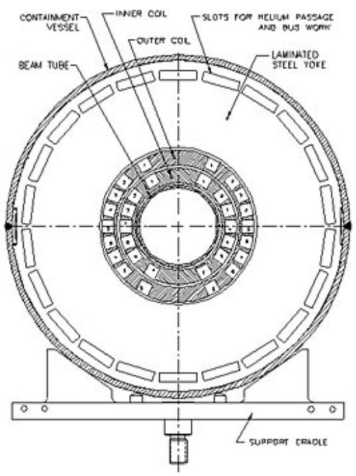




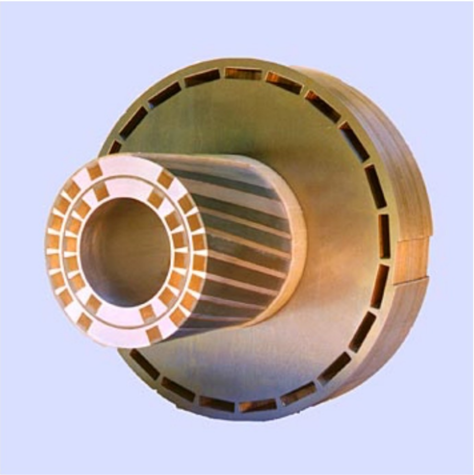
Orbit deviation drops inversely proportionally
to beam energy.
The resulting orbit excursion is considerably less
than in Steffen’s snake!

But helical magnets have intrinsically non-linear fields,
thus the effect on particle dynamics should be carefully
evaluated.
(Betatron tune shift, beta-function distortions)

RHIC and AGS Helical Snake Pictures



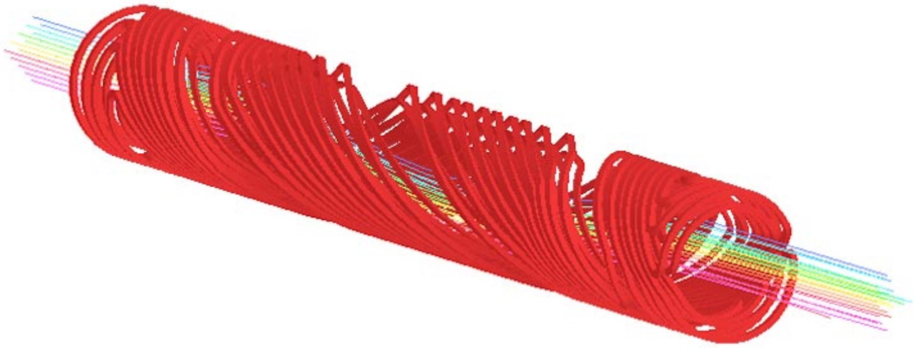
Cross section drawing of a helical magnet for RHIC. The inner coil diameter is 80 mm.



Photograph of a section cut from a prototype RHIC helical dipole magnet.

Strong magnetic field is preferrable. It makes Snakes more compact and minimizes the orbit excursion

Magnet aperture must be large enough to accommodate large orbit excursion (~2cm) At RHIC injection energy (25 GeV).



Drawing of the two-layer coils for the AGS Snake. Note that the pitch of the helix for the center portion of the coils is less than for the outer portion of the coils.

Superconducting magnet technology, using wire with NbT superconductor, has been used for building 4T helical magnets for RHIC Snakes and spin rotators, as well as for one AGS Snake.

Some Key Points Re-Iterated

- Siberian Snake is an amazing device allowing polarization preservation while crossing numerous and numerous spin resonances during beam acceleration.
- Most efficient use of Snakes is in pairs (even number), with proper distribution of an accelerator ring. Proper selection of the snake axis angle ensures spin tune 0.5
- Even with Snakes: be careful about depolarization, there are higher order resonances, “Snake” resonances. Larger beam energies require larger number of the Snakes
- Spin rotator is very important device: most of experiments done on colliders want longitudinally polarized beam at collision points.
- Practical realization of Snakes and rotators depends on the energy of a particular accelerator.
  Dipole, and helical dipole based Snakes are proper choice at higher energies (>20 GeV);
  while solenoidal based snakes at lower energies (<20 GeV))

Thank you for your attention!

Additional Reading

In addition to materials listed in the course, for this particular topic (Snakes and Rotators) following materials are recommended:

1. Handbook of Accelerator Physics and Engineering,
sections 2.6.3 (Spin Rotators and Siberian snakes) and 7.2.18 (Spin Manipulation).

2. "Siberian Snakes in high-energy accelerators", S.R.Mane, Yu.M.Shatunov, K.Yokoya, Journal of Physics G: Nuclear and Particle Physics, 31 (2005) R151.

3."Helical Spin Rotators and Snakes", V.Ptitsyn and Yu.M.Shatunov, NIM A 398 (1997), p.126.

4. S.Y.Lee, NIM A 306 (1991), p.1.